LES simulation of vertical mixing

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January 22, 2007

1. Explicit LES for stratified flows
   - Özgökmen et al. (2007)
   - Pacanowski and Philander (1981)
   - Mellor and Yamada (1982)

2. Some comments about implicit LES
   - Basic ideas
   - Basic example

Özgökmen's approach (1/2)

- Use the Smagorinsky model for calculating turbulent viscosity coefficients:

$$\nu_T = \nu_{smag} \equiv (c_s \delta)^2 \sqrt{\sum_{i,j} d_{ij}^2},$$

where δ is the filter scale and $d_{ij} = (\frac{\partial u_j}{\partial x_i} + \frac{\partial u_i}{\partial x_j})/2$ is an element of the strain rate tensor.

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- Modify Smagorinsky's vertical viscosity to take into account the effect of stratification:

$$\nu_{ozg} \equiv f_{ozg}(Ri)\nu_{smag}.$$

Özgökmen et al. (2007)'s approach (2/2)

- The function f_{ozg} is a dimensionless function of the Richardson number. It goes to one when there is no stratification and to zero when stratification is important. It is defined as:

$$f_{ozg}(Ri) = \begin{cases} 1 & \text{for } Ri < 0, \\ \sqrt{1 - \frac{Ri}{Ri_c}} & \text{for } 0 \leq Ri < Ri_c, \\ 0 & \text{for } Ri > Ri_c, \end{cases}$$

where Ri_c is the critical Richardson number, typically $Ri_c = 0.25$.

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- This way to regulate vertical mixing is quite arbitrary and other approaches could be used as well.

Pacanowski and Philander's parameterization (1/2)

- They proposed the following low order turbulence closure:

$$\nu_{paca} = \frac{K_u^0}{(1 + \beta_M Ri)^{\alpha_M}} + K_u^*,$$

where $K_u^0 \approx 10^{-2}$ m^{2}s^{-1} is a sort of upper bound on the vertical eddy viscosity value while $K_u^* \approx 10^{-4}$ m^{2}s^{-1} is a much smaller background value, although larger than the molecular viscosity.

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- This suggests the following definition of the weighting function:

$$f_{paca}(Ri) = \begin{cases} 1 & \text{for } Ri < 0, \\ \frac{1}{(1+\beta_M Ri)^{\alpha_M}} & \text{for } Ri \geq 0, \end{cases}$$

  which exhibits the same behaviour as f_{ozg}.

Pacanowski and Philander's parameterization (2/2)

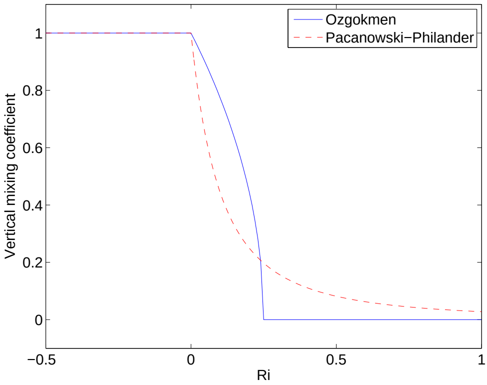

Figure: Comparison of the functions $f_{ozg}(Ri)$ and $f_{paca}(Ri)$ used to take the effect of stratification into account.

What about Mellor-Yamada level 2.5?

- This is a higher order turbulence closure where the eddy viscosity is expressed as:

$$\nu_{MY} = lqS_u,$$

where l and q are the turbulence length and velocity scales, respectively. The stability function S_u is a dimensionless function of l, q and the Brunt-Väisälä frequency.

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- This suggests the following definition of the weighting function:

$$f_{MY}(Ri) = \begin{cases} 1 & \text{for } Ri < 0, \\ \frac{lqS_u}{K_u^0} & \text{for } Ri \geq 0, \end{cases}$$

where K_u^0 is again an upper bound on the vertical eddy viscosity.

Implicit LES: Basic ideas

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- It is based on the use of non-linear "smart" discretizations that incorporate the SGS physics into the numerics
- According to its partisans, implicit LES avoids commutation errors and does not decouple the physical model from the numerics. The model and the numerics cannot be decoupled unless the flow is fully resolved.

Basic example (1/2)

- 1D advection equation:

$$\frac{\partial c}{\partial t} + w\frac{\partial c}{\partial z} = 0,$$

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- The upwind advection term can be rewritten as a centered (non dissipative) advection term plus a diffusion term:

$$\frac{c_i^{n+1} - c_i^n}{\Delta t} + w_i \frac{c_{i+1}^n - c_{i-1}^n}{2\Delta z} = \frac{1}{2}\frac{w_i}{\Delta z}\Delta z^2 \frac{c_{i+1}^n - 2c_i^n + c_{i-1}^n}{\Delta z^2}.$$

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- Let's try this:

$$\frac{c_i^{n+1} - c_i^n}{\Delta t} + w_i \left[f(Ri) \frac{c_i^n - c_{i-1}^n}{\Delta z} + (1 - f(Ri)) \frac{c_{i+1}^n - c_{i-1}^n}{2\Delta z} \right] = 0.$$

In that case, the amount of upwinding is maximal when there is no stratification and decreases with increasing stratification.

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  In that case, the amount of upwinding is maximal when there is no stratification and decreases with increasing stratification.
- That discretization is equivalent to the following one:

$$\frac{c_i^{n+1}-c_i^n}{\Delta t}+w_i\frac{c_{i+1}^n-c_{i-1}^n}{2\Delta z}=\frac{1}{2}f(Ri)\frac{w_i}{\Delta z}\Delta z^2\frac{c_{i+1}^n-2c_i^n+c_{i-1}^n}{\Delta z^2}.$$